

## SURFACE QUALITY OF A WORK MATERIAL INFLUENCE ON VIBRATIONS IN A CUTTING PROCESS

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*(Received 23 October 2000, and in final form 21 March 2001)*

The problem of stability in the machining processes is an important task. It is strictly connected with the final quality of a product. In this paper we consider vibrations of a tool-workpiece system in a straight turning process induced by random disturbances and their effect on a product surface. Basing on experimentally obtained system parameters we have done the simulations using one degree of freedom model. The noise has been introduced to the model by the Langevin equation. We have also analyzed the product surface shape and its dependence on the level of noise.

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### 1. INTRODUCTION

The quality of a final surface in a cutting process is of a natural interest of industry and technology. Grabec [1, 2] and Gradisek with co-workers [3] analyzed a simple orthogonal cutting model and found that, chaotic conditions of tool-workpiece system, given by appropriate system parameters, are possible. As they have demonstrated, the appearance of such chaotic conditions can have crucial effect on the stability of cutting process. The chaotic vibrations also were investigated experimentally by Tansel and others in [4]. On the other hand instabilities of cutting process have been known for long time as a chatter phenomenon [5, 6, 7]. The mechanism their appearance includes the

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nonlinear self-excitation in the cutting process, which leads to vibrations with larger value of amplitude, beyond the admissible limit. One of the source of instabilities can be identified in the roughness of a material initial surface, which introduces randomness of the material resistance during the dynamic process. Wiercigroch and Cheng [8] have investigated the influence of noise on the orthogonal cutting system. In their analysis they started from the spectral representation of stochastic process. Nevertheless, the most common treatment of dynamical processes influenced by noise is the Fokker-Plank approach [9, 10, 11]. Because of some considerable difficulties which it meets by solving in higher dimensions as well as for numerical reasons it could be transformed to the corresponding Langevin equations [10, 11]. Here, following the papers [11, 12, 13, 14], we use the stochastic Langevin equations with an additive white noise and solve the dynamic equations of examined system. Previous articles [8, 11, 13] devoted to cutting process in presence of noise, focused rather on the problem of dynamics and possibilities of bifurcations induced by the random disturbances. Interestingly, Wiercigroch and Cheng [8] and later Przystupa and Litak [11], investigating orthogonal cutting process with two degrees of freedom, claimed that in some conditions weak noise can even stabilize the chaotic attractor. On the other hand paper [14] deals with the reconstruction of the system dynamics from the stochastic time series. In their treatment the cutting process was assumed to be deterministic but the measured data were influenced by noise coming from the measurement procedure.

Our paper is also a contribution to a complicated problem of dynamics of a cutting process, but we focus on the final quality of a product surface, and in

this context, in the stability of process. Note, in Fig. 1, the shape of an initial surface of a cut workpiece. The shape has numerous imperfections which we will model by random deviations from the ideal cylindrical surface. Adopting a simple one degree-of-freedom model of regenerative cutting [15] we included the effect of a previous pass of a straight turning process by a time delay term [16].

## 2. DETERMINISTIC MODEL OF CUTTING PROCESS

The physical model of a straight turning process, corresponding to our experimental system, is presented in Fig. 2. Here we have introduced the following notations:  $v_f$  is a relative velocity between the tool and the workpiece,  $h_0$  is an assumed initial while  $h$  an actual cutting depth;  $w$  is a principal axis of relative vibrations;  $y$  indicates the direction normal to the axis of a workpiece symmetry,  $\kappa$  is a tool cutting edge angle;  $k$  and  $c$  are the stiffness and damping of the system respectively;  $n$  denotes a rotational velocity of a workpiece;  $f$  denotes the direction of feed in a straight turning;  $m$  is the effective mass of the system.

The main vibration in  $w$  direction, perpendicular to the cutting edge, (Fig. 2) and to be precise we should analyze vibrations as well as cutting force in  $w$  direction. However we are interested in the final surface profile given by the time history of  $y$  and not by the actual profile  $w$ . To analyze vibration in  $y$  we have done simultaneous projecting vibrations and forces into  $y$  direction. Thus the deterministic equation of motion of dynamical system, projected on a normal (to final surface) direction  $y$ , can be written as follows [15]:

$$\ddot{y} + 2\tilde{n}\dot{y} + p^2y = \frac{K}{m}g_y(h, v_f), \quad (1)$$

where  $p$  is a natural frequency of free vibrations of the workpiece  $p^2 = k/m$  while  $\tilde{n} = c/m$ . Nonlinearities, appearing in that system, are included in the  $g_y$  function [1, 2, 3, 8, 15]:

$$g_y(h, v_f) = \left[ c_2 \left( \left| \frac{v_f}{v_0} \right| - 1 \right)^2 + 1 \right] \left[ c_3 \left( \frac{h}{h_0} - 1 \right)^2 + 1 \right] \frac{h}{h_0} \Theta(h) \text{Sgn}(v_f). \quad (2)$$

Cutting depth  $h$  and relative velocity  $v_f$  are defined [14, 15]:

$$h = h_0 + y(t') - y(t), \quad v_f = 1 - \frac{\dot{y}}{v_0}. \quad (3)$$

$\Theta(h)$  and  $\text{Sgn}(v_f)$  correspond to step functions: Heaviside and sign functions respectively and  $v_0$  is the linear velocity of a rotational motion of a workpiece during a steady cutting process.

$t'$  is the time of a previous pass:

$$t' = t - \Delta t, \quad (4)$$

where  $\Delta t$  is a workpiece revolution time during machining. The shape of a nonlinear function  $g_y$  (Eq. 2) dependent on  $h$  and  $v_f$  is presented in Fig. 3. Note the two-dimensional surface  $g_y = g_y(h, v_f)$  was plotted only for positively defined  $h$ . In case of negative  $h$ , the force on the left hand side of Eq. 1, is zero because of the contact loss between the tool and a workpiece. The sudden sign change of a cutting force in a function of relative velocity  $v_f$  is due to a friction phenomenon between the tool and a chip.

Our model (Eqs. 1-3) with one degree-of-freedom is a serious simplification of a physical situation. However our aim is not the comprehensive description of a cutting process. Here we want to concentrate on particular aspects of it. In spite of simplicity of the examined model still allows the chatter vibrations to be generated due to the nonlinearities in of the cutting force  $g_y(h, v_f)$  as it

was shown in paper [15]. In our model chatter is generated by a combination of the friction phenomenon between the tool and chips, and the impact of a tool after loosing its contact with a workpiece. Warmiński and others [15] examined the second pass of the orthogonal cutting process by using similar model. The results obtained there have indicated that such model can lead to periodic, quasi-periodic as well as chaotic vibrations due to the initial harmonic modulation of the machined surface.

For a numerical calculation purpose we have written the equations (Eqs. 1-4) in discrete way introducing the constant time step  $\tau$ :

$$\begin{aligned} t_{r+1} &= t_r + \tau, \\ y_{r+1} &= y_r + v_r \tau, \\ v_{r+1} &= v_r + (-2\tilde{n}v_r - p^2 y_r + \frac{K}{m} g_{yr}) \tau, \end{aligned} \tag{5}$$

where  $t_r$  is a sampling discrete time after  $r$  time steps. The function  $g_{yr}$  should be expressed:

$$\begin{aligned} g_{yr} &= g_y(h_r, v_{fr}), \\ h_r &= h_0 + y_s - y_r, \\ v_{fr} &= 1 - \frac{v_r}{v_0}, \end{aligned} \tag{6}$$

where  $r$  and  $s$  are natural numbers. The time difference between  $y_r$  and  $y_s$  coordinates;  $\Delta t = (r - s)\tau$  relates to the time of workpiece revolution (Eq. 4). The system parameters obtained from the experiment are following:  $p = 785$  rad/s,  $m = 12.1$ kg,  $K = 620$  N,  $h_0 = 1.5 \times 10^{-3}$  m,  $f = 0.1 \times 10^{-3}$  m/rev,  $2\tilde{n} = 190$  1/s,  $v_0 = 0.1$  m/s  $\kappa = 70^\circ$  and  $c_2 = 0.5$ ,  $c_3 = 1.55$  are cutting process constants derived from [7, 15].

### 3. INFLUENCE OF NOISE

Most of real dynamical processes are disturbed by the random signal. In case of cutting process they come through the roughness of the initial surface (Fig. 1). Other sources of such disturbances can be found in a spontaneous breaking of chips and the couplings of the tool and the workpiece to other dynamic parts of the experimental standing.

To describe the stochastic system we introduce to the model a random component by an additive white noise of Gaussian distribution [10, 11, 12, 13, 14]. Usually stochastic dynamic systems are investigated by using the Fokker-Planck equation [8, 9, 10, 11]. One dimension version of it reads:

$$\frac{\partial}{\partial t}P(y, t) = \frac{\partial}{\partial y} [\nu(y)P(y, t)] + D\frac{\partial^2}{\partial y^2}P(y, t), \quad (7)$$

where  $D$  denotes the diffusion coefficient,  $\nu(x)$  is, in general, the non-linear drift term (driving force) and  $P(x, t)$  the probability distribution function. As solving Fokker-Planck equation meets some considerable difficulties [10, 11] we have transformed it into the corresponding Langevin equation:

$$\dot{y} = z(y) + g\Gamma(t), \quad (8)$$

where  $g\Gamma(t)$  is a Gaussian distributed random 'force' with the strength  $g$  and  $\Gamma(t)$  is assumed to satisfy:

$$\begin{aligned} \langle \Gamma(t) \rangle &= 0, \\ \langle \Gamma(t), \Gamma(t') \rangle &= 2\delta(t - t'), \end{aligned} \quad (9)$$

where the brackets denote an average over the probability distribution function. Starting with the definitions of the drift  $\nu$  and diffusion  $D$  coefficients obtained by the Kramers-Moyal expansions in the derivation of Fokker-Planck

equation from a Chapman-Kolmogorov equation [17], we can find the relation between  $\nu(y)$  and  $D$  of the Fokker-Plank equation (Eq. 7) with  $z(y)$  and  $g$  of the Langevin equation (Eq. 8). Thus the Langevin equation can be finally expressed by drift and diffusion terms of the initial Fokker-Plank equation as:

$$\dot{y} = \nu(y) + \sqrt{D}\hat{\Gamma}(t) \quad (10)$$

For the actual numerical calculation we have discretized form:

$$y(t + \tau) - y(t) = \int_t^{t+\tau} dt' v(t') + \sqrt{D} \int_t^{t+\tau} dt' \Gamma(t') \approx \nu(t)\tau + \sqrt{D}\hat{\Gamma}(t), \quad (11)$$

where

$$\hat{\Gamma}(t) = \int_t^{t+\tau} dt' \Gamma(t') \quad (12)$$

is a superposition of Gaussian distributed random numbers which again are of a Gaussian form. Namely:

$$\hat{\Gamma}(t) = a\omega(t). \quad (13)$$

In our case the average value of  $\omega$  has been chosen as  $\langle \omega \rangle = 0$  while its variance as  $\langle \omega^2 \rangle = 2$ , respectively. From the integration of the Gaussian (Eq. 12) the coefficient  $a$  (Eq. 13) depends on the time integration step  $\tau$  via  $a = \sqrt{\tau}$ . Equation 8 can be, in general, solved by higher order algorithms like Runge-Kutta one [18]. However here, for simplicity, we have limited our discussion to the simplest algorithm of Euler type. Thus, the final form of the Langevin equation in the lowest order perturbation, suitable for numerical integration is given by the following expression:

$$y_{r+1} = y_r + \nu(y_r)\tau + \sqrt{D}\sqrt{\tau}\omega(t_r). \quad (14)$$

Here we analyzed the one-dimensional version of the Langevin and Fokker-Plank equations. The similar discussion on the  $m$ -dimensional stochastic Langevin equation and its relation to the corresponding Fokker-Plank equation can be found in [14].

In our model, Eq. 9 for  $y_r$  has to be supplemented by the rest of equations (Eqs. 5,6) for discrete time  $t_r$  and velocity  $v_r$  which is now substituted by the corresponding drift term  $\nu_r$ :

$$\nu_{r+1} = \nu_r + (-2\tilde{n}v_r - p^2y_r + \frac{K}{m}g_{yr})\tau. \quad (15)$$

Using the above procedure (Eqs. 4,5 and Eq. 14-15) we have done the simulations for a constant time step  $\tau = 0.741 \times 10^{-4}$  s, corresponding to the workpiece revolution time  $\Delta t = 0.741 \times 10^{-1}$  s and a number of diffusion constants values  $D$ . Figures 4 a-d show the time histories of  $y$  of initial 3 seconds of cutting work for experimentally identified system parameters (Sec. 2). For a deterministic system ( $D = 0$ , Fig. 4a) we observe the stable cutting process with no vibrations. Figures 4b, c relate to cutting process in presence of noise. The diffusion constant values in these figures are  $D = 10^{-5}$  and  $D = 10^{-4}$  respectively. One can see easily the presence of small vibrations (Fig. 4b), which are growing with increasing of the noise level (Fig. 4c). Obviously, such vibrations have a significant effect on the quality of a workpiece surface. It is shown in Fig. 5, where the error shape of a surface is plotted as a function of a workpiece rotation angle after 3 s of cutting work.

Modulation of shape caused by random disturbances depends on a noise level. Both: input and output random signals can be easily measured by standard deviations. For various  $D = 10^{-6}$ ,  $10^{-5}$  and  $10^{-4}$  we have got the following values of standard deviations:  $\sigma = 3.79 \times 10^{-7}$  m,  $1.21 \times 10^{-6}$  m



and  $4.16 \times 10^{-6}$  m. In Figs. 6a and b we have compared the input and output random signals for one of the above cases ( $D = 10^{-5}$ ). Figure 6a shows the distribution of Gaussian disturbances of input noise  $\omega$  (Eqs. 8,9) while Fig. 6b corresponds to the errors of workpiece shape after cutting. In Fig. 6b deviation from the normal probability distribution is caused by nonlinear dynamics of the cutting process. To quantify the system answer we have used standard deviations of the output signal  $\sigma$  as a function of diffusion constant  $D$ . It is plotted in the logarithmic scale in Fig. 7. We have checked that  $\sigma(D)$  can be scaled as a square root as far as noise level is low while for stronger noise its effect on the fluctuations of  $y$  is stronger.

#### 4. SUMMARY AND CONCLUSIONS

We have considered vibrations of a tool-workpiece system in a straight turning process induced by random disturbances and their effect on a product surface. Using a single degree-of-freedom model we have focused on the combined effects of the friction nonlinearities and tool-workpiece contact loss. We have noticed that for large enough level of noise, the tool and a workpiece start to vibrate due to random forcing. Such excitation can interact with a complex dynamics of the system leading to process non-stability and, in the end, to much worse final quality of the machined product. In case of a relatively small level of noise (a small value of the diffusion constant  $D$ ) the surface shape error scales as a square root of  $D$ . For a higher value of a noise level the shape error is proportional to  $D$ . Clearly, in a straight turning the initial surface roughness influence the quality of a final product. This is the principal result of our paper, which lead to a conclusion that one has to prepare workpiece

which initial surface satisfies the appropriate criteria.

## ACKNOWLEDGEMENTS

The work has been partially supported by Polish State Committee for Scientific Research (KBN) under the grant No. 126/E-361/SPUB/COST/T-7/DZ 42/99. We would like to thank the organizers of 3<sup>rd</sup> International Symposium "Investigation of Nonlinear Dynamic Effects in Production Systems" 26-27 September 2000 in Cottbus (Germany) for giving two of us (J.L. and R.R.) opportunity to present this work. We would like to thank the unknown referee for valuable comments and Dr W. Przystupa for helpful discussions.

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## FIGURE CAPTIONS

Figure 1. Experimental standing with a cast iron workpiece.

Figure 2. Physical model of a straight turning process.

Figure 3. Nonlinear function  $g_y(h, v_f)$  versus cutting relative velocity.

Figure 4. Time histories of  $y$  for various values of diffusion constants (a)  $D = 0$ , (b)  $D = 10^{-5}$ , (c)  $D = 10^{-4}$ .

Figure 5. Shape error as a function of workpiece rotation angle after 3s of cutting.

Figure 6. Distribution probabilities of random input (a) and output  $y$  (b) signals of the model for  $D = 10^{-5}$ .

Figure 7. Standard deviation  $\sigma$  as a function of a diffusion constant  $D$ .

## FIGURES



FIG. 1.

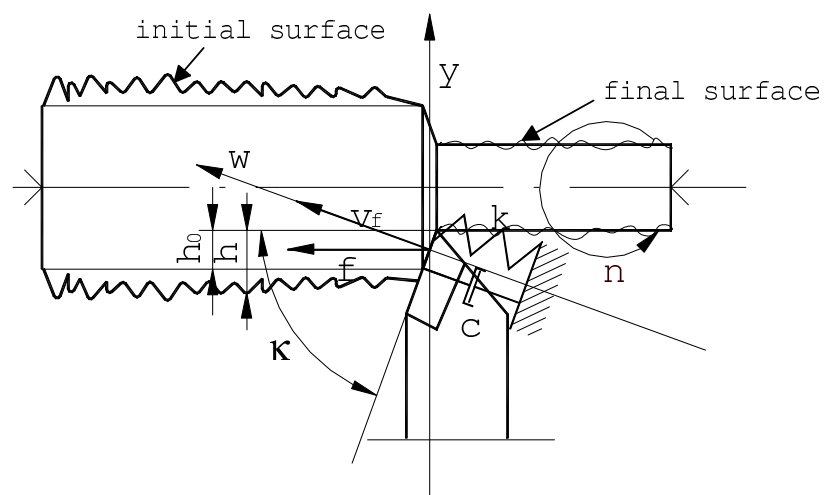


FIG. 2.

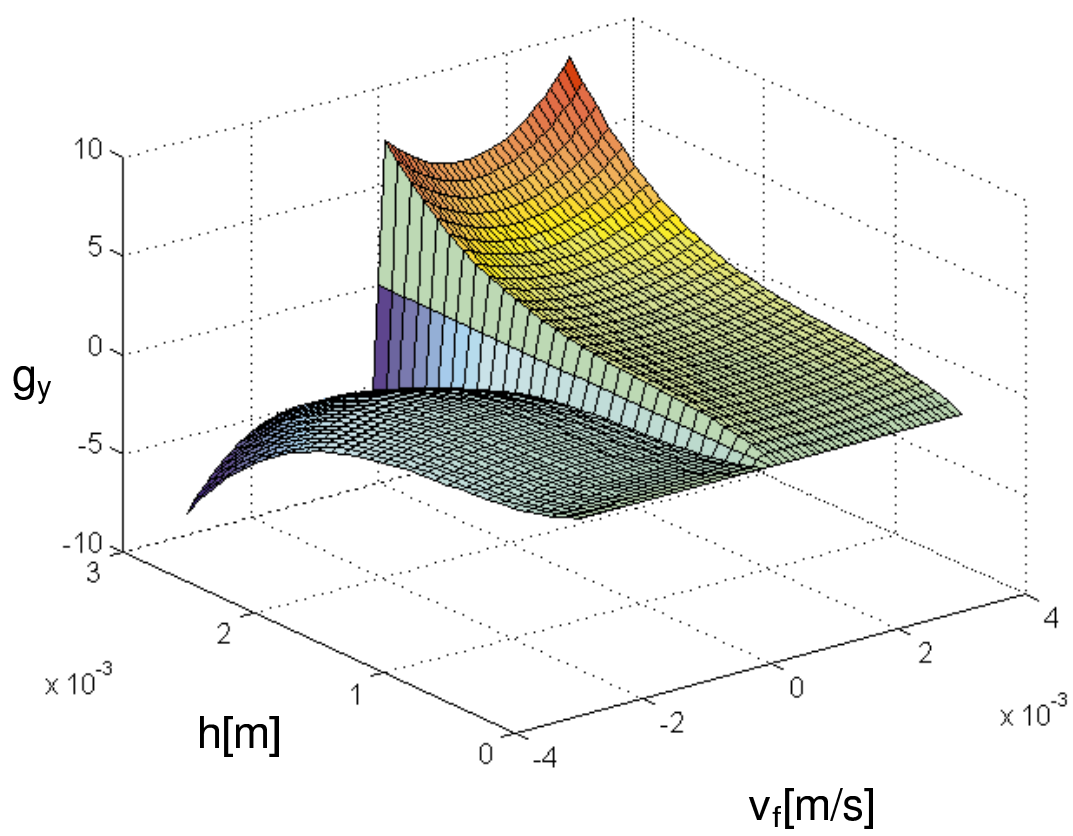


FIG. 3.

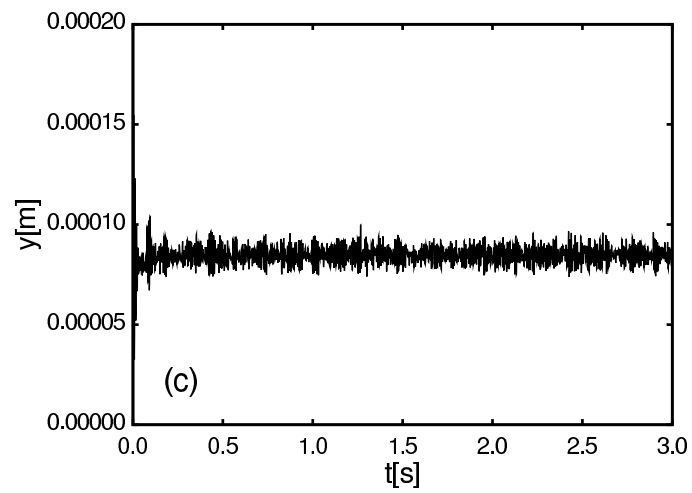
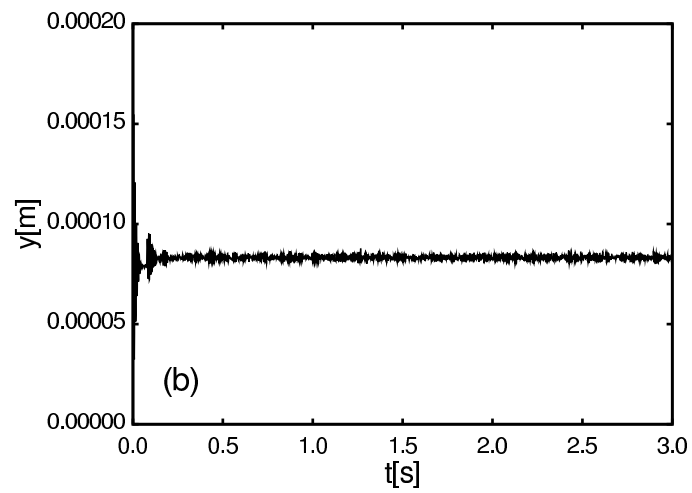
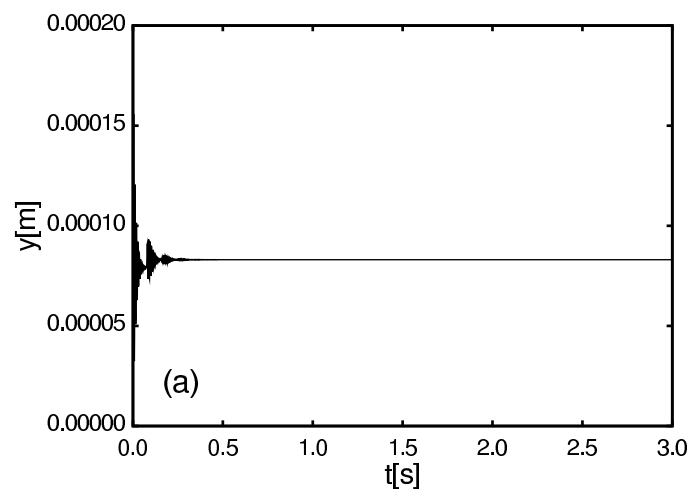


FIG. 4.



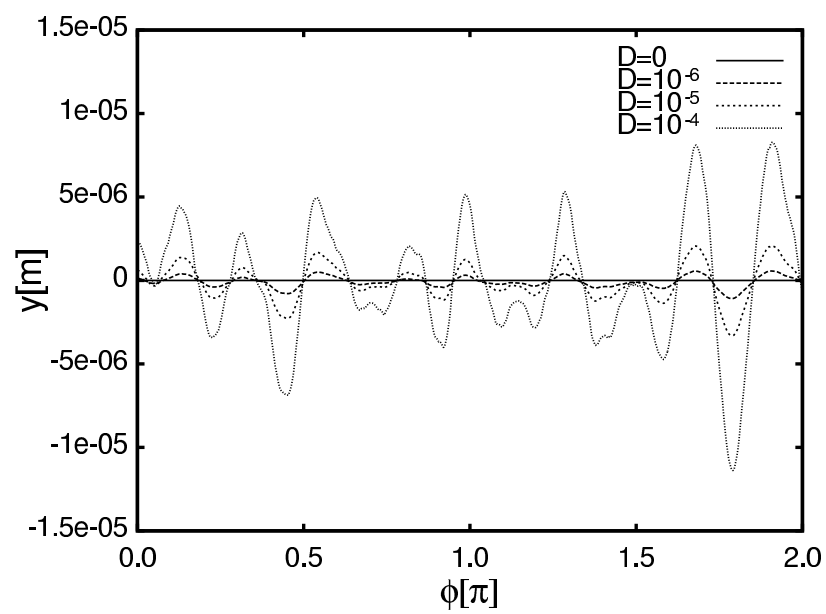


FIG. 5.

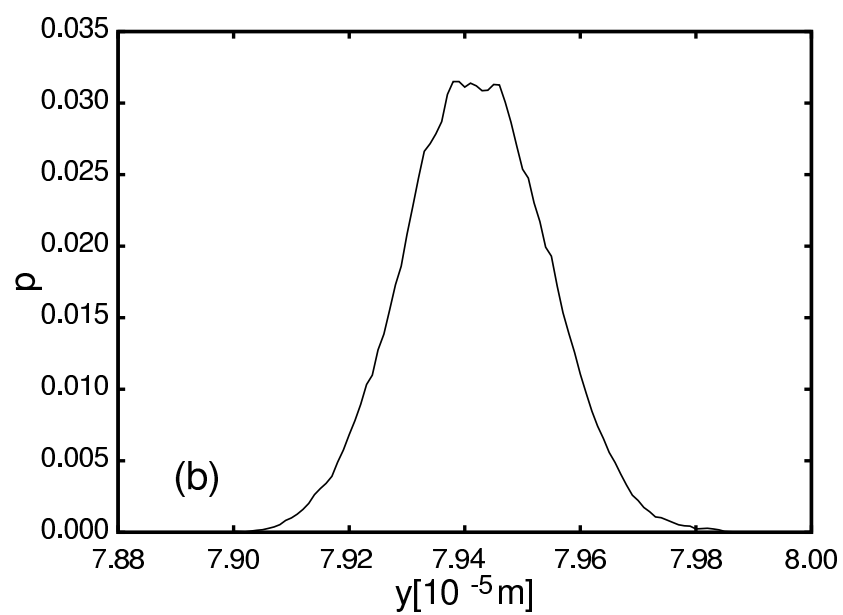
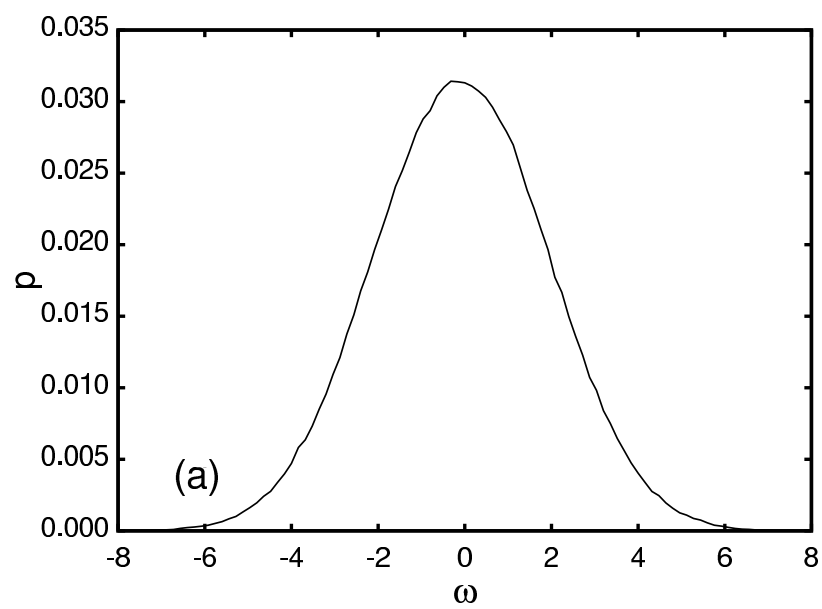


FIG. 6.

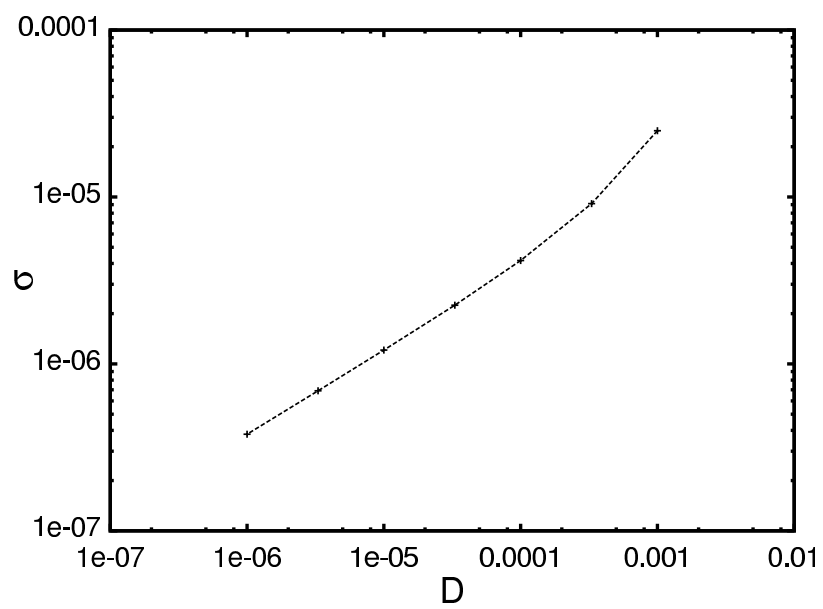


FIG. 7.